Probability Models

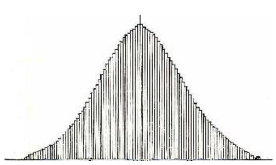
Suppose you are playing a game that has only two possible outcomes (succeed/fail) whose probabilities are constant and independent of previous games. Such a game is called a **Bernoulli Trial** and has a very simple probability model. A common example of a Bernoulli trial is a coin toss.

We are generally interested in only two types Bernoulli trials. The first, called the **Geometric Model**, is the solution to question of the form "What is the probability that I have to play the game *n* times before my first win". The second, called the **Binomial Model**, is the solution to questions of the form "What is the probability that I win *x* times after playing *n* games". These two models can be described by a simple set of equations.

Geometric Models are completely specified by *p*, the probability of success. This means that you can know everything there is to know about a Geometric model from *p* (i.e. p is the only variable). The - answer to the question above is [$(1-p)^{n-1}p$]. The first term in this equation is the probability of failing n-1 times in a row and the final term is the probability of succeeding on the n^{th} trial. The expected value for the number of trials until the first win is 1/p with a standard deviation of $\sqrt{\frac{1-p}{p^2}}$.

Binomial models are defined by n, the number of trials, and p, the probability of success. The equation that describes the binomial method looks intimidating but is not that bad if you know what the different parts do. The solution to the question above has the form $\binom{n}{x}p^x q^{n-x}$. The term in parentheses is the combination term and takes into account all the possible ways you can have x successes out of n trials. The second term is the probability of winning *x* times and the last term is the probability of failing the rest of the time. The expected number of wins is *np* with a standard deviation of $\sqrt{np(1-p)}$.

Calculations involving the Binomial model can be tedious. So for large values of n, the binomial model can be approximated by the normal model. The only condition you have to check before using this approximation is that you expect more than 10 successes and 10 failures. Note that when using this approximation you can only give probabilities for intervals of values.



This is a graph of the binomial model for a large number of trials. Note how it is very nearly normal. However, the data is still discrete, not continuous.

When working with these, or any other probability models, keep these points mind:

-knowing the derivation of a formula makes it easier to remember and understand

-check all assumptions and conditions to make sure the model you plan on using is relevant

For Bernoulli trials check for: two outcomes, constant probabilities, and independence

-make a picture (chart, tree diagram, Venn…) whenever you get stuck

-be aware of any special conditions, exceptions, and limits to the models you use

-study the different types of questions the models are designed to answer